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A statistical characterization of differences and similarities of aggregation functions

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Abstract

As information fusion is becoming relevant to many quantitative fields of science and economics, aggregation functions have been deeply studied in order to investigate their analytical properties. More recently, researchers are experiencing the practical application of aggregation functions. Along the problem of fitting an appropriate function to empirical data, there is the need of studying the behavior of models. In this paper we focus on the statistical study of the output values as a way to compare and choose among different alternatives by simulation.

Key words: Aggregation Function, Information Fusion, Statistics.

1 Introduction

In many applications, there is a need for aggregating several input values into a single output value. The aggregation is often related to quantitative information. This task becomes central to several problems in fields such as Physics, Computer Science, Engineering, Social Sciences, Economics and Finance [1]. In the recent years, a relevant research effort has been produced in order to characterize the framework of aggregation functions.

With respect to this last meaning, an aggregation function is a mapping $F : R^n \to R$. Generally input and output values belong to some ratio scale. Therefore, without any loss of generality we can rescale all values to the unit interval I. So an aggregation function can be defined as $F : I^n \to I$.

In particular, researchers paid attention to conjunctive functions (e.g. t-norms), disjunctive functions (e.g. t-conorms), and compensatory functions (e.g. means, OWA, uninorms) in order to characterize them by analytical properties. More recently, attention has been paid to how to fit models to empirical data [2, 3]. In this context becomes relevant to study the distribution of output values and how it is related to input values.

For instance, conjunctive functions concentrate values in the lower part of I, whilst disjunctive functions will concentrate values in the upper part. Compensatory functions will concentrate values in the central part. Therefore, looking at the output distributions we can verify if two aggregation functions have a different behavior or they follow a similar pattern from a statistical point of view.

Despite the evidence that statistics provide robust and meaningful tools for characterizing aggregation functions, this approach has never been deeply investigated. Many questions can arise by studying the output distribution from a statistical point of view. In this paper we will focus on pairwise comparing aggregation functions on the basis of their output distributions in order to answer questions such as: Are statistical differences among conjunctive (disjunctive) and compensatory functions? Do the number of data points to aggregate as any effect on the output distributions? Do the number of variables to aggregate emphasize differences between functions? Does the input distribution change the behavior of outputs?. Answering these questions provide insights useful to practitioners in choosing between alternatives with substantial statistical differences.

The remainder of this paper is organized as follows: in Section 2 we provide some preliminaries in order to get a common basis for discussion; in Section 3 we state the theoretical output distributions for some of the functions defined previously; in Section 4 we develop the experimental study by simulating several cases and comparing them in order to give answer to some of the questions stated above; in Section 5 we outline conclusions and future directions.

2 Preliminaries

Given the scale I and the order n, all possible aggregation functions are functions being developed within the hypercube I^{n+1}. In this context, an n-ary *aggregation function* is a mapping $F_{(n)} : I^n \to I$. The number of arguments may not be known in advance. Therefore an aggregation function is in general a mapping $F : \bigcup_{n\in\mathbb{N}} I^n \to I$, where $F|_{I^n} = F_{(n)}\ \forall n \in \mathbb{N}$, so that a general aggregation function can be regarded as a family $F = \big(F_{(n)}\big)_{n\in\mathbb{N}}$. For the sake of simplicity, in the following sections we will mostly refer to binary functions. Moreover, since the interval I plays the role of *scale*, we can focus on $I \equiv [0, 1]$ without any loss of generality. Thus, we will consider the class of mappings $F_{(n)} : [0, 1]^n \to [0, 1]$ and in particular $F_{(2)} : [0, 1] \times [0, 1] \to [0, 1]$.

There are some basic properties able to characterize a generic function as an aggregation function [4].

Boundary conditions. For any aggregation function there are two boundary

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conditions to meet, namely

$$F_{(n)}(0,\dots,0) = 0 \quad \text{and} \quad F_{(n)}(1,\dots,1) = 1 \tag{1}$$

Both the conditions preserve the domain bounds, so that the minimal inputs lead to the minimal output value, whilst the maximal inputs lead to the maximal output value. This property becomes a natural assumption within the context of *multi-criteria decision making* (i.e. mcdm) problems. Indeed, we expect that the aggregation of completely unsatisfactory (negative, false) criteria is itself completely unsatisfactory (negative, false). Similarly, we expect that the aggregation of fully satisfactory (positive, true) criteria is itself fully satisfactory (positive, true).

Monotonicity. The monotonicity of aggregation functions is generally considered non-decreasing with respect to each variable (marginal monotonicity), so that

$$F_{(n)}(x_1,\dots,x_n) \geq F_{(n)}(x'_1,\dots,x'_n) \qquad \forall x_i \geq x'_i,\ i = 1..n \tag{2}$$

The non-deincreasingness reflects the assumption in mcdm scoring problems that better input values should lead to an overall better aggregated value. In other words, better scores cannot provide a worse overall score.

Identity when unary. This simple property states that

$$F_{(1)}(x) = x \qquad \forall x \in I \tag{3}$$

that is the aggregated result should not differ from the input value in the trivial case of one argument.

Beside these fundamental properties, there are other useful properties. In particular, the notion of strength provides a partial ordering of aggregation functions.

Strength. This property is related to the comparison of two aggregation functions $F_{(n)}$ and $F'_{(n)}$, so that $F_{(n)}$ is stronger than $F'_{(n)}$ ($F_{(n)} \succcurlyeq F'_{(n)}$) iff

$$F_{(n)}(x_1,\dots,x_n) \geq F'_{(n)}(x_1,\dots,x_n) \qquad \forall x_i \in I,\ i = 1..n \tag{4}$$

Duality. For each aggregation function F, there exists a dual function F' defined as

$$\hat{F}(x_1,\dots,x_n) = 1 - F(1 - x_1,\dots,1 - x_n) \tag{5}$$

When met, this property allows to evaluate the overall behavior of an aggregation function, so that it is assumed to provide higher or lower aggregated values in comparison to another function. Moreover, aggregation functions can be partially ordered by strength, so that this property provides a criterion for classifying three main categories of aggregation functions, namely the t-norms, t-conorms and averages, discussed below.

The class of aggregation functions is very large. There are many other properties able to characterize some groups of aggregation functions. More detailed overview of aggregation functions and properties can be found in references [1]. Instead, we will focus on noticeable example of aggregation functions, in particular examples belonging to the class of t-norms, t-conorms and compensatory functions.

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2.1 Triangular norms

A triangular norm (i.e. *t-norm*) is an aggregation function $T : [0,1]^2 \rightarrow [0,1]$ that is symmetric, associative, with neutral element 1. The dual function $S : [0,1]^2 \rightarrow [0,1]$, that is also symmetric and associative, but with neutral element 0, is called a triangular conorm (i.e. *t-conorm*). It is possible to prove that t-norms have absorbent element 0, while t-conorms have absorbent element 1.

To the class of t-norms belong many functions commonly used in applicative domains. Some examples are reported in Table 1.

Minimum	$T_M(x_1, x_2) = \min(x_1, x_2)$
Product	$T_P(x_1, x_2) = x_1 \cdot x_2$
Lukasiewicz	$T_L(x_1, x_2) = \max(x_1 + x_2 - 1, 0)$
Drastic	$T_D(x_1, x_2) = \begin{cases} x_1 & x_2 = 1 \\ x_2 & x_1 = 1 \\ 0 & otherwise \end{cases}$
Yager $(p > 0)$	$T_Y(x_1, x_2) =$ $\max\left(1 - [(1 - x_1)^p + (1 - x_2)^p]^{\frac{1}{p}}, 0\right)$
Schweizer and Sklar $(q > 0)$	$T_S(x_1, x_2) =$ $1 - [(1 - x_1)^q + (1 - x_2)^q - (1 - x_1)^q (1 - x_2)^q]^{\frac{1}{q}}$
Frank $(s > 0, s \neq 1)$	$T_F(x_1, x_2) = \log_s \left(1 + \frac{(s-1)^{x_1} + (s-1)^{x_2}}{s-1}\right)$

Table 1: Some examples of t-norms

Although the set of t-norms is not totally ordered (at least it has not been proven yet), it is possible to prove that any generic t-norm T is constrained, more precisely

$$T_D \preccurlyeq T \preccurlyeq T_M \tag{6}$$

Thus, t-norms are conjunctive. However, it holds

$$T_D \preccurlyeq T_L \preccurlyeq T_P \preccurlyeq T_M \tag{7}$$

For each t-norm there exists a dual counterpart in the set of t-conorms. For instance, Table 2 reports the (standard) dual t-conorms of Table 1.

In the case of t-conorms, it is possible to verify

$$S_M \preccurlyeq S \preccurlyeq S_D \tag{8}$$

and

$$S_M \preccurlyeq S_P \preccurlyeq S_L \preccurlyeq S_D \tag{9}$$

Therefore t-conorms are disjunctive.

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Maximum	$S_M(x_1,x_2) = \max(x_1,x_2)$
Probabilistic Sum	$S_P(x_1,x_2) = x_1 + x_2 - x_1x_2$
Lukasiewicz	$S_L(x_1,x_2) = \min(x_1+x_2,1)$
Drastic	$S_D(x_1,x_2) = \begin{cases} x_1 & x_2 = 0 \\ x_2 & x_1 = 0 \\ 1 & otherwise \end{cases}$
Yager $(p > 0)$	$S_Y(x_1,x_2) = \min\left([x_1^p + x_2^p]^{\frac{1}{p}}, 1\right)$
Schweizer and Sklar $(q > 0)$	$S_S(x_1,x_2) = [x_1^q + x_2^q - x_1^q x_2^q]^{\frac{1}{q}}$
Frank $(s > 0, s \neq 1)$	$S_F(x_1,x_2) = 1 - \log_s\left(1 + \frac{(s-1)^{1-x_1}+(s-1)^{1-x_2}}{s-1}\right)$

Table 2: Some examples of t-conorms

2.2 Compensatory functions

An aggregation function M is said compensatory if it holds the compensatory property, that is

$$\min \preccurlyeq M \preccurlyeq \max \tag{10}$$

Examples of compensatory aggregation are presented in Table 3.

The class of compensatory functions is not limited to the means. This is the case of the exponential compensatory functions. Other functions can be built as composition of t-norms and t-conorms, for instance the convex-linear compensatory functions are defined as

$$L_\gamma^{T,S}(x_1,\ldots,x_n) = (1-\gamma)T(x_1,\ldots,x_n) + \gamma S(x_1,\ldots,x_n) \tag{11}$$

In general, aggregation functions are not associative; so they are all functions presented above.

3 Output distributions

We can relate the cumulative distribution function of output values (i.e. cdf) to the structure of levels, as

$$Pr(F(x_1,...,x_n) \leqslant z) = 1 - \mu_F(z) \tag{12}$$

where $\mu_F(z)$ is the strict α-cut measure at level z. Indeed the strict α-cut of F, collects all points $\mathbf{x} \in I^n$, such that $F(\mathbf{x}) > z$, as depicted in Fig.3.

Instead, the probability density function (pdf) is defined as

$$Pr(F(x_1,...,x_n) = z) = -\frac{d}{dz}\mu_F(z) \tag{13}$$

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Arithmetic Mean	$AM(x_1,\ldots,x_n) = \frac{1}{n}\sum\limits_{i=1}^{n} x_i$
Geometric Mean	$GM(x_1,\ldots,x_n) = \left(\prod\limits_{i=1}^{n} x_i\right)^{\frac{1}{n}}$
Quadratic Mean	$QM(x_1,\ldots,x_n) = \sqrt{\frac{1}{n}\sum\limits_{i=1}^{n} x_i^2}$
Harmonic Mean	$HM(x_1,\ldots,x_n) = \left(\frac{1}{n}\sum\limits_{i=1}^{n} \frac{1}{x_i}\right)^{-1}$
Quasi-arithmetic Mean ($\alpha \in \overline{\mathbb{R}}$)	$QAM(x_1,\ldots,x_n) = \left(\frac{1}{n}\sum\limits_{i=1}^{n} x_i^{\alpha}\right)^{\frac{1}{\alpha}}$
Exponential Compensation ($\gamma \in [0,1]$)	$Z_\gamma(x_1,\ldots,x_n) = \left(\prod\limits_{i=1}^{n} x_i\right)^{1-\gamma} \cdot \left(1 - \prod\limits_{i=1}^{n}(1-x_i)\right)^{\gamma}$

Table 3: Some examples of compensatory functions

Minimum. Alpha cuts are squares whose sides become smaller by level increasing, and whose measure is $\mu_F(z) = (1-z)^2$, so $CDF_{T_M}(z) = 2z - z^2$ and $PDF_{T_M}(z) = 2(1-z)$.

Product. In this case, the area of the α-cut at level z is $\mu_F(z) = \int\limits_z^1 \frac{z}{x}dx = 1 + z(\log z - 1)$. Thus, the product cdf and pdf are $CDF_{T_P}(z) = z(1 - \log z)$ and $PDF_{T_P}(z) = -\log z$.

Lukasiewicz t-norm. The alpha-cut measure at $z = 0$ is $1/2$. Indeed, $\mu_F(z) = \frac{1}{2}(1-z)^2$ and $CDF_{T_L}(z) = 1 - \frac{1}{2}(1-z)^2$ and $PDF_{T_L}(z) = \frac{1}{2}\delta(z) + (1-z)$.

Drastic t-norm. The drastic t-norm is 0 almost everywhere, except on the sides $x = 1$ and $y = 1$. Therefore: $CDF_{T_D}(z) = 1(z)$ and $PDF_{T_D}(z) = \delta(z)$.

The cdf and pdf of some functions above are plotted in Fig.3

Levels of dual functions are symmetric as shown by Fig.3 in the case of product and probabilistic sum. The symmetry between dual functions suggests a general rule for computing cdf and pdf, as stated below.

Proposition 3.1 *Let $\hat{F}(\cdot)$ be the dual function of $F(\cdot)$, the following equality stands:*

$$CDF_{\hat{F}}(z) = 1 - CDF_F(1-z) \quad \text{and} \quad PDF_{\hat{F}}(z) = PDF_F(1-z)$$

Proof. Since $\hat{F}(\mathbf{x}) = 1 - F(1-\mathbf{x})$, α-cuts of $\hat{F}$ are obtained as depicted in Fig.3. Thus,

$$\mu_{\hat{F}}(z) = 1 - \mu_F(1-z)$$

and this concludes the proof.

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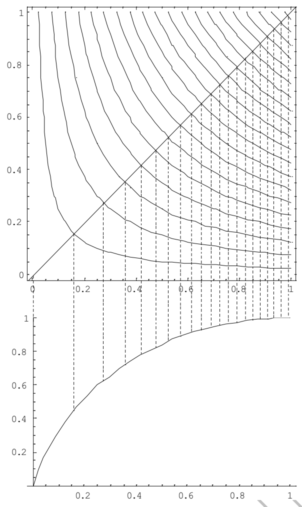

Figure 1: Relationships between function levels and output cdf

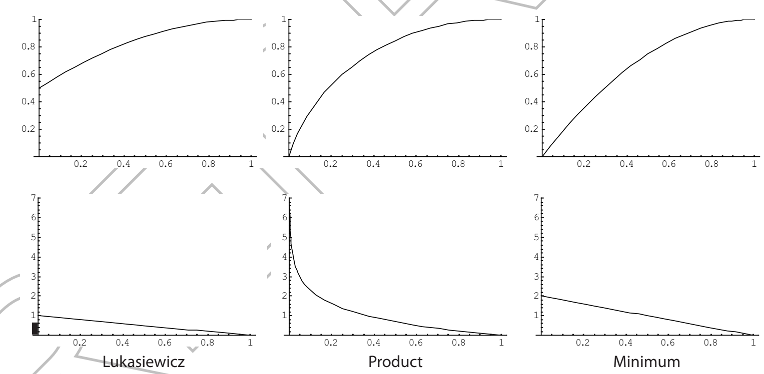

Figure 2: The cdf (top) and pdf (bottom) of noticeable t-norms

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Similarly, we can obtain the distribution functions of some averages. With respect to two input variable, we can determine the output distribution of arithmetic mean and quadratic mean (see Fig.3)

Arithmetic Mean.

$$CDF_{AM}(z) = \begin{cases} 2z^2 & 0 \leq z < \frac{1}{2} \\ 1 - 2(1-z)^2 & \frac{1}{2} \leq z \leq 1 \end{cases} \tag{14}$$

$$PDF_{AM}(z) = \begin{cases} 4z & 0 \leq z < \frac{1}{2} \\ 4(1-z) & \frac{1}{2} \leq z \leq 1 \end{cases} \tag{15}$$

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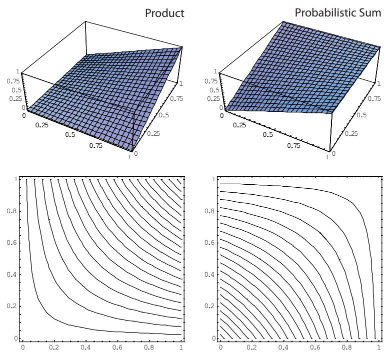

Figure 3: Comparison of dual functions product and probabilistic sum

Quadratic Mean.

$$CDF_{QM}(z) = \begin{cases} \frac{1}{2}z^2\pi & 0 \le z < \frac{1}{\sqrt{2}} \\ \sqrt{2z^2-1} + z^2\left(\frac{\pi}{2} - 2\arctan\sqrt{2z^2-1}\right) & \frac{1}{\sqrt{2}} \le z \le 1 \end{cases} \quad (16)$$

$$PDF_{QM}(z) = \begin{cases} z\pi & 0 \le z < \frac{1}{\sqrt{2}} \\ 2z\left(\frac{\pi}{2} - 2\arctan\sqrt{2z^2-1}\right) & \frac{1}{\sqrt{2}} \le z \le 1 \end{cases} \quad (17)$$

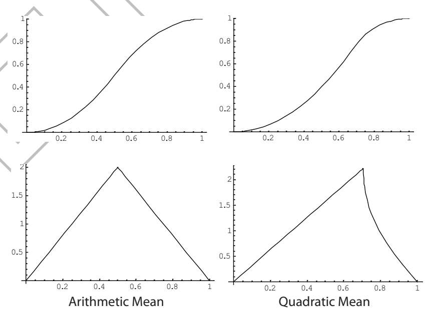

Figure 4: The cdf (top) and pdf (bottom) of arithmetic and quadratic mean.

The measure $\mu_F(z)$ involves the computation of multidimensional integrals, that sometimes can be difficult or impossible to compute analytically. In this case, the cdf can be computed numerically, by approximation or montecarlo simulation.

4 Differences and similarities in the aggregation operators outputs

In this section we analyze the impact that changes in the distribution of the input values and the number of arguments produce on the distribution of the aggregation function. We also pay attention to the effect of increasing the sample size, that is, the

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number of points we are aggregating, since we develop the analysis by using simulation. This type of study is a very important question since the original distribution over the inputs has obvious effects on the values of the aggregation function: for instance, the result for a conjunctive function with uniform inputs will differ from the result using an input distribution that concentrates values in the lower part of I. Also the number of inputs is an important source of variability since the distribution of the aggregation functions changes when considering more arguments. On the other hand, the sample size is key point since the larger the sample size is the better is our information about the behavior of the outputs and, thus, we are in a better position to see the differences between outputs. As an additional analysis, not following the same procedure but straightly related to this one is the fitting of the empirical distribution of the outputs to a given theoretical distribution.

Therefore, our interest in this section is to establish a procedure that helps the practitioner in deciding which aggregation function to choose. And the answer that we are giving is based on the statistical significance of the differences between two (or more) aggregation functions. So that, when we are not able to deduce significant differences between the outputs of two aggregation functions, we can make our choice knowing that the relevance for the data is not important.

It should be underlined, from the beginning of this procedure, that we are focusing on the experimental results of the outputs and not on the theoretical basis of the inputs. This is very important since we are not stating that two different aggregation functions are equal in any sense, but we are empirically checking that, despite the original differences in the distribution of the inputs, the outputs does not provide statistically significant differences. Thus, from a statistical point of view, we could consider the two aggregation functions to be indifferent regarding the distribution of the data. Moreover, increasing the sample size will always ease the possibility to differentiate the effect of different aggregation operators.

In order to clarify these ideas we have followed a standard procedure, that we show below and then we illustrate it with some examples.

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4.1 Procedure

Procedure: Comparing aggregation outputs distributions.

1. Fix the number k of aggregation functions to compare.

2. Fix the sample size.

3. Fix the number n of arguments (inputs).

4. Fix the probability distributions over input values.

5. Generate a sample of input values x_{lj}, for $j = 1, \ldots, k$ and $l = 1, \ldots, n$.

6. Choose the aggregation functions to evaluate: $F_1, \ldots, F_k$.

7. Obtain the output values $Y_1, \ldots, Y_k$ by making $Y_j = F_j(x_{1j}, \ldots, x_{nj})$, for $j = 1, \ldots, k$.

8. Pose the question: 'Can the values of $Y_1, \ldots, Y_k$ can be considered to have the same distribution?'. If $k = 2$, perform a Mann-Whitney-Wilcoxon Test over the

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outputs Y_1, Y_2. Obtain a p-value. If $k > 2$, perform a Kruskal-Wallis Test over the outputs $Y_1, \ldots, Y_k$. Obtain a p-value.

9. If the p-value is greater than or equal to the prefixed significance level, then the conclusion is: 'The outputs can be considered to have the same distribution'. Else, the conclusion is: 'The outputs are significantly different'.

10. End.

We have to remark than when solving a statistical test, the conclusion is either we have found evidences to reject our question (null hypothesis) or we do not have found evidences to reject the hypothesis. Hence, a substantial nuance is that we are not properly saying that the outputs are similar but they can considered to be similar.

It is a must to emphasize that we are not saying that different aggregation operators applied to the *same* inputs produce similar results. We are not referring to fixed individuals, because in that case the differences is immediate. On the contrary, our conclusion is statistical in the following sense: we are concluding that when applying different aggregation operators on a population of inputs with a certain distribution, our results can considered to be statistically similar, that is, in general, most of our outputs will have similar behavior.

We also observe that the sample size is prefixed at the beginning of the procedure. As we briefly pointed out before, the sample size is crucial in determining the differences, because the greater is the sample size the more powerful is the test to appreciate differences.

A final important remark concerns the test we are using. In case we have a Gaussian distribution over the data $Y_1, \ldots, Y_k$ we could use other types of tests, like T-tests/ANOVA for instance, but it is important to note that we are more focused on distribution rather than equality of means, that is the hypothesis tested by T-tests/ANOVA.

4.2 Examples

Obviously, the number of possible cross-combinations over input distributions/aggregation functions/number of arguments/sample sizes is impossible to handle. Thus, we show some very illustrative examples. In all of them we have perfomed several simulations (up to 10000 iterations per each one) and them we have analyzed the overall results. Other parameters in the model are: number of input arguments, sample size (number of items to be aggregated) and input distributions. Instead of providing the exhaustive results of all simulations, we prefer to explain the general results.

Example 1. Consider two t-norms. A symmetry of this analysis can be translated to the case of t-conorms, due to the dual property described in Section 2. It is very important to take into account the dominance relationships because Mann-Whitney-Wilcoxon test uses ranks, so that, it will produce significative differences when comparing dominated outputs. We provide examples of comparison of two dominated t-norms. In **case A** we test minimum versus product. In **case B** we test minimum versus Łukasiewicz. Also we provide examples in which we compare two t-norms without

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dominance relationship. In **case C** we test product and Yager. In this case, we see how, for certain values of the parameter p of Yager t-norm, the results of the aggregation can be considered to follow the same pattern. All cases are constructed from uniform inputs.

Example 2. Consider two averages. By using uniform inputs we observe how analytically different aggregation operators (see Section 2) produce either clearly different outputs (**case D**: geometric mean versus arithmetic mean) or not so different outputs (**case E**: quadratic mean versus arithmetic mean).

Example 3. In this case we variate the sample size. As it was expected, the greater is the sample size the better situation to differentiate the outputs of different aggregation functions. We perform several trials increasing and decreasing the sample size used in examples 1 and 2 and we check how the different/similar relationships varies. In particular, we mention the conclusions for cases C, D and E. In **case C**, we observe that, for very close values of parameter p of Yager t-norm, differences do not increase when increasing the sample size. In **cases D** and E, we find that the smaller is the sample size the less important is the choosing of the average by the practitioner, that is, if the practitioner is aggregating a small quantity of points the election of the average is not so relevant.

Example 4. Now we study how changes in the number of arguments can affect the results. By increasing the number of arguments we see how certain aggregation functions change their behavior, with respect to that analyzed in cases C and E. In **case C**, we note that increasing the number of arguments also increase very quickly the differences between product and Yager t-norms. But when we compare two Yager t-norms with close values for the parameter p we find that the differences are slightly decreasing when the number of arguments increase. As in **case E**, the differences also increase when increasing the arguments but in a much slower way, because of compensation properties of averages.

Example 5. Finally, we analyze what happens when we use different distributions than uniform on the inputs. We use Beta distributions to move the concentration of the values to different areas on interval I. We observe that the relationships obtained in examples 1 and 2 can variate depending on which values are more weighted by the aggregation operator and when the values are more concentrated. Actually in **case A** differences between minimum and product t-norms become greater when the Beta distribution is unimodal and concentrates values in one subinterval of I (for instance, when using B(1,10), B(2,2), or B(10,1)). The reason is that this concentration of input values weaken the compensations that uniformity produced among extreme values. On the other hand if the Beta distribution is bimodal (as B(.5,.5)) we observe that this compensation becomes greater and, thus, it is much more difficult to appreciate differences between minimum and product. This also occurs when values are concentrated in an opposite way, that is, when inputs in one argument accumulate in an opposite subinterval than the second argument (B(10,1) and B(1,10), for instance). Similar behavior occurs in **case C**. This inclines us to think that the reason of this behavior is not dominance between t-norms (it is valid in case A but not in case C) but the reinforcement property for t-norms. Thus, when weighting more on some subinterval of I the

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reinforcement property becomes more important to differentiate t-norms. In **case E** the behavior is just opposite to cases A and C. Obviously, since we are now averaging, the compensation that produces the operator becomes less effective when the values are concentrated in some subinterval of I and, then, is much easier to differentiate the outputs. That is why the behavior is complementary to the one showed by t-norms. In summary, the reinforcement/compensatory property, accentuate differences between output distributions when inputs concentrate in some regions of the unit square.

5 Conclusion and future works

In this paper we have performed an statistical analysis on similarities and differences within three main groups of aggregation functions: conjunctive, disjunctive and compensatory functions. We have obtained several conclusions described in the examples, that, in some cases, allow the practitioner to make his decision about the aggregation function that needs to use. In other cases, theoretical outputs distributions lead us to clearly differentiate between the simulated outputs.

Our future work is focused on analyzing the behavior of outputs under different probability distribution and the study of the role that compensatory, conjunction and disjunction properties play on this behavior. Moreover, we are also planning to provide a procedure to check the goodness-of-fit of a given output to a known distribution. Finally, another open problem is to study what is the real effect of different aggregation functions on the same set of input data. We plan to develop all this in a forthcoming paper.

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